Section 2.1 Techniques for Finding Derivatives (Minimum problems: 1 – 19 odds, 21, 25, 29, 33, 43, 47, 49, 53, 57)

We are going to learn how to calculate derivatives with rules that will allow us to "quickly" compute derivatives without the need of the limit definition of the derivative that we learned in Chapter 1.

We should get used to some common derivative notations:

| Function | Sample function | Symbolic way to ask you to find the derivative | Symbolic way to represent answer |
|---|---|---|---|
| $f(x) =$ | $f(x) = 6x^2 - 7x + 1$ | $\frac{d}{dx}f(x)$ | $f'(x)$ |
| $g(t) =$ | $g(t) = \frac{t+6}{t-g}$ | $\frac{d}{dt}g(t)$ | $g'(t)$ |
| $y = some\ function\ of\ x$ | $y = 3^{4x^2}$ | $\frac{d}{dx}y$ | Either $y'$ or $\frac{dy}{dx}$ |
| $z = some\ function\ of\ t$ | $z = 2t - 7$ | $\frac{d}{dt}z$ | Either $z' or$ $\frac{dz}{dt}$ |

**POWER RULE:**

The Power Rule is the first rule that we will learn to compute derivatives without using limits. It is useful when finding the **derivative** of a function that is raised to the nth **power**.

## Here are a few representations of the Power Rule (multiply exponent by coefficient and lower exponent by 1)

- $\boldsymbol{f(x) = ax^n \text{ then } f'(x) = nax^{n-1}}$
- $\boldsymbol{y = ax^n \text{ then } \frac{dy}{dx} = nax^{n-1} \text{ or } y' = nax^{n-1}}$

Here are a few examples of the power rule:

| Function | Algebra to find derivative | Derivative |
|---|---|---|
| $f(x) = 5x^4$ | $f'(x) = 4 * 5x^{4-1}$ | $f'(x) = 20x^3$ |
| $y = 7x^{-2}$ | $\frac{d}{dx}y = -2 * 7x^{-2-1} = -14x^{-3}$ | $\frac{dy}{dx} = -\frac{14}{x^3}$ |
| $y = 6x^{3/2}$ | $y' = \frac{3}{2}6x^{\frac{3}{2}-1} = 9x^{\frac{1}{2}}$ | $y' = 9x^{\frac{1}{2}}$ or $y' = 9\sqrt{x}$ |
| $t = 2x$ | $\frac{d}{dx}t = 1 * 2x^{1-1} = 2x^0 = 2$ | $\frac{dt}{dx} = 2$ |

**The derivative of any 1st power term is its coefficient.**

$$\boldsymbol{f(x) = ax^1 \quad then \;\; f'(x) = a}$$

Here are a few examples that show the derivative of any 1st power term is its coefficient.

| Function | Rewrite | Algebra to find derivative | Derivative |
|---|---|---|---|
| $y = 7x$ | $y = 7x^1$ | $y' = 1 * 7x^{1-1}$<br>$= 7x^0$<br>$= 7 * 1$ | $y' = 7$ |
| $g(t) = 9t$ | $g(t) = 9t^1$ | $g'(t) = 1 * 9t^{1-1}$<br>$= 9t^0$<br>$= 9 * 1$ | $g'(t) = 9$ |
| $f(x) = -3x$ | $f(x) = -3x^1$ | $f'(x) = 1 * -3x^{1-1}$<br>$= -3x^0$<br>$= -3 * 1$ | $f'(x) = -3$ |

**The derivative of any constant term is 0.**

$$\boldsymbol{f(x) = a \quad then \;\; f'(x) = 0}$$

Here are a few examples that show the derivative of any 0th power term is 0.

| Function | Rewrite | Algebra to find the derivative | Derivative |
|---|---|---|---|
| $y = 7$ | $y = 7x^0$ | $y' = 0 * 7x^{0-1}$ <br> $= 0x^{-1}$ <br> $= 0$ | $y' = 0$ |
| $g(t) = 9$ | $g(t) = 9t^0$ | $g'(t) = 0 * 9t^{0-1}$ <br> $= 0t^{-1}$ <br> $= 0$ | $g'(t) = 0$ |
| $f(x) = -3$ | $f(x) = -3x^0$ | $f'(x)$ <br> $= 0 * -3x^{0-1}$ <br> $= 0x^{-1} = 0$ | $f'(x) = 0$ |

**Sum Rule for derivatives:**
(Allows us to take the derivative of each term in a sum individually when we find a derivative.)

$$\boldsymbol{\frac{d}{dx}(f(x)+g(x))=f'(x)+g'(x)}$$

Here is an example of how to use the sum rule:

| Function | Algebra to find the derivative | Derivative |
|---|---|---|
| $y=7x^2+6x$ | $\frac{dy}{dx}=2*7x^{2-1}+6$ <br> $=14x+6$ | $\frac{dy}{dx}=14x+6$ |

**Difference Rule for derivatives:**
(Allows us to take the derivative of each term in a difference individually when we find a derivative.)

$$\boldsymbol{\frac{d}{dx}(f(x)-g(x))=f'(x)-g'(x)}$$

Here is an example of how to use the difference rule:

| Function | Algebra to find the derivative | Derivative |
|---|---|---|
| $f(x)=9x^5-11$ | $f'(x)=5*9x^{5-1}-0$ <br> $=45x^4$ | $f'(x)=45x^4$ |

Many of the homework problems in this section will need to be rewritten before we find the derivative.

Example: Rewrite and find the derivative: $y = \frac{2}{x}$

$y = \frac{2}{x}$

$y = \frac{2}{x^1} = 2x^{-1}$

This is rewrite that will allow us to use the Power Rule

$y = 2x^{-1}$

$y' = -1 * 2x^{-1-1}$

$y' = -2x^{-2}$

Answer: $y' = -\frac{2}{x^2}$

Example: Rewrite and find the derivative: $f(x) = (3x + 5)(2x - 3)$

$f(x) = (3x + 5)(2x - 3)$

$f(x) = 6x^2 - 9x + 10x - 15$

$f(x) = 6x^2 + 1x - 15$

This is rewrite that can be used to find the derivative

$f(x) = 6x^2 + 1x - 15$

$f'(x) = 2 * 6x^{2-1} + 1 - 0$

$f'(x) = 12x + 1$

Answer: $f'(x) = 12x + 1$

Example: Rewrite and find the derivative: $f(x) = \frac{4x^3+6x^2}{2x}$

$f(x) = \frac{4x^3+6x^2}{2x}$

$f(x) = \frac{4x^3}{2x} + \frac{6x^2}{2x}$

$f(x) = 2x^2 + 3x$

This is the rewrite we can use to find the derivative

$f(x) = 2x^2 + 3x$

Answer: $f'(x) = 4x + 3$

Example: $f(x) = 3x^2 - 4x + 5$; $x = 2$

a) Find the slope of the tangent line to the graph of the function for the given value of x.

b) Find the equation of the tangent line to the graph of the function for the given value of x.

a) Find the slope of the tangent line to the graph of the function for the given value of x.

The slope of a tangent line at a value of x can be found by substituting that value of x into the DERIVATIVE

$f'(x) = 6x - 4$

slope of tangent line when $x = 2$: $f'(2) = 6(2) - 4 = 8$

Answer: $m = 8$

b) Find the equation of the tangent line to the graph of the function for the given value of x.

We know the slope and the x-coordinate of a point. We first need to find the y-coordinate by plugging in x = 2 into the ORIGINAL FUNCTION.

$y = f(2) = 3(2)^2 - 4(2) + 5 = 12 - 8 + 5 = 9$

Point $(2,9)$ $slope \quad m = 8$

Equation of line:

$y - 9 = 8(x - 2)$

$y - 9 = 8x - 16$

Answer: $y = 8x - 7$

Minimum homework: 1 – 19 odds, 21, 25, 29, 33, 35, 37, 41, 45

#1-20: Use the Power rule to find the derivative of each function (write each answer with positive exponents)

1) $f(x) = 3x^2 + 4x - 7$

2) $g(x) = 2x^5 - 5x^3 + 3x - 4$

$Answer\ \ g'(x) = 10x^4 - 15x^2 + 3$

Minimum homework: 1 – 19 odds, 21, 25, 29, 33, 35, 37, 41, 45

3) $y = 5x^3 + 3x + 1$

4) $y = 6x^3 + x^2 + 21$

$Answer$: $\frac{dy}{dx} = 18x^2 + 2x$

Minimum homework: 1 – 19 odds, 21, 25, 29, 33, 35, 37, 41, 45

5) $y = 9x^2 + 5x - 4$

6) $y = 12x^2 - 3x - 4$

$Answer\ \ y' = 24x - 3$

Minimum homework: 1 – 19 odds, 21, 25, 29, 33, 35, 37, 41, 45

7) $f(x) = 18$

8) $h(x) = -14$

$Answer\colon\ h'(x) = 0$

Minimum homework: 1 – 19 odds, 21, 25, 29, 33, 35, 37, 41, 45

9) $y = 3\sqrt{x}$

10) $y = 12\sqrt[3]{x}$

$Answer\ \ y' = \frac{4}{\sqrt[3]{x^2}}$

Minimum homework: 1 – 19 odds, 21, 25, 29, 33, 35, 37, 41, 45

11) $g(x) = 6\sqrt{x}$

12) $f(x) = 14\sqrt{x}$

$Answer$: $f'(x) = \frac{7}{\sqrt{x}}$

Minimum homework: 1 – 19 odds, 21, 25, 29, 33, 35, 37, 41, 45

13) $f(x) = 3x^{2/3}$

14) $g(x) = 10x^{1/2}$

$answer \quad g'(x) = \frac{5}{\sqrt{x}}$

Minimum homework: 1 – 19 odds, 21, 25, 29, 33, 35, 37, 41, 45

15) $f(x) = x^{1/3}$

16) $f(x) = x^{\frac{1}{5}}$

$answer$: $f'(x) = \frac{1}{5\sqrt[5]{x^4}}$

Minimum homework: 1 – 19 odds, 21, 25, 29, 33, 35, 37, 41, 45

17) $y = \frac{3}{x^2}$

18) $y = \frac{5}{x^3}$

$\frac{dy}{dx} = \frac{-15}{x^4}$

Minimum homework: 1 – 19 odds, 21, 25, 29, 33, 35, 37, 41, 45

19) $f(x) = \frac{-3}{x}$

20) $f(x) = \frac{-5}{x}$

$answer \quad f'(x) = \frac{5}{x^2}$

Minimum homework: 1 – 19 odds, 21, 25, 29, 33, 35, 37, 41, 45

#21-32: Clear the parenthesis and then find the derivative of each function.

21) $y = (2x + 3)(3x - 4)$

22) $y = (3x - 4)(5x - 8)$

$Answer\ \ y' = 30x - 44$

Minimum homework: 1 – 19 odds, 21, 25, 29, 33, 35, 37, 41, 45

23) $f(x) = (x - 2)(3x - 4)$

24) $y = (x - 5)(3x^2 + 7)$

25) $f(x) = (x^2 + 3x + 2)(3x - 5)$

26) $f(x) = (3x^2 + 6x - 2)(4x + 1)$

$answer$: $f'(x) = 36x^2 + 54x - 2$

Minimum homework: 1 – 19 odds, 21, 25, 29, 33, 35, 37, 41, 45

27) $g(t) = (2t - 1)(3t + 5)$

28) $g(t) = (3t^2 + 5t)(2t + 1)$

29) $y = 3x^2(2x^2 + 6x - 4)$

30) $y = 4x^3(3x^2 + 7x - 5)$

$answer$: $y' = 60x^4 + 112x^3 - 60x^2$

Minimum homework: 1 – 19 odds, 21, 25, 29, 33, 35, 37, 41, 45

31) $f(x) = (5x^2)(4x)$

32) $f(x) = (7x^2)(6x)$

#33-40: Rewrite the problem without a fraction and find the derivative of each function.

33) $f(x) = \frac{3x^2+6x}{2x}$

34) $f(x) = \frac{4x^3+6x^2+10}{2x}$

$answer\colon\ f'(x) = 4x + 3 - \frac{5}{x^2}$

Minimum homework: 1 – 19 odds, 21, 25, 29, 33, 35, 37, 41, 45

35) $y = \frac{x^2+2x}{x}$

36) $y = \frac{x^2+4x}{x}$

$answer\ \ y' = 1$

Minimum homework: 1 – 19 odds, 21, 25, 29, 33, 35, 37, 41, 45

37) $f(x) = \frac{24x^2+12x+60}{12}$

38) $f(x) = \frac{25x^2+5x+15}{5}$

$f'(x) = 10x + 1$

Minimum homework: 1 – 19 odds, 21, 25, 29, 33, 35, 37, 41, 45

39) f(x) = $\frac{5x^2+6x+1}{x^2}$

40) f(x) = $\frac{3x^2-x+1}{x^2}$

#41-44:

a) Find the slope of the tangent line to the graph of the function for the given value of x.

b) Find the equation of the tangent line to the graph of the function for the given value of x.

41) $f(x) = 3x^2 + 6x - 2; \quad x = 2$

42) $f(x) = 2x^2 - 6x - 2; \quad x = 3$

$answer$ a) $m = 6$ b) $y = 6x - 20$

Minimum homework: 1 – 19 odds, 21, 25, 29, 33, 35, 37, 41, 45

43) $f(x) = 9x^3 - 12x^2 + 5;\ \ x = 3$

44) $f(x) = 10x^3 - 5x^2 + 3;\ \ x = 4$

#45-48:

a) Find all values of x where the tangent line is horizontal

b) Find the equation of the tangent line to the graph of the function for the values of x found in part a.

45) $f(x) = 3x^2 + 6x + 2$

46) $f(x) = 2x^2 - 8x + 7$

$answer\ \ a)\ x = 2$

b) $y = -1$

47) f(x) = $-4x^2 + 24x - 9$ 48) $f(x) = -2x^2 + 12x + 3$